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I. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

$31\times\frac{1}{2}\sqrt{2}+1=22.9201$.

Hence, we can put in 32 layers ; 16 layers of 576 in each layer and 16 layers of 529 in each. There still remains space enough for one more layer.

$23\times\frac{1}{2}\sqrt{3}+1=20.918$.

Hence, in this layer, we can put 3 rows of 24 balls each and 24 rows of 24 and 23 alternately, or 636 in the whole layer.

$$\begin{aligned}\therefore\ 16\times576&=\ 9216\\ 16\times529&=\ 8464\\ 1\times636&=\ \ \ 636\\ \hline \text{Total}&=18316\end{aligned}$$

II. Solution by MARTIN H. SPINKS, Wilmington, Ohio.

Take the bottom layer and the rows in equilateral triangular form. The distance between the rows is .866 inch. The number of rows$=1+(23\div.866)=1+26=27$.

We then have 14 rows, 24 balls each, or 336 balls, and 13 rows, 23 balls each, or 299 balls each.

The bottom layer contains 336+299 or 635 balls. In the next layer we have 14 rows of 23 balls each or 322 balls, 13 rows of 24 balls each or 312 balls, in all 634 balls.

Distance between layers=.8162 inch.

Number of layers$=1+(23\div.8162)=28+1=29$.

Space left$=24-(1+28\times.8162)=.1464$ inch.

$\therefore$ We have 15 layers, 635 balls each, or 9525 balls
and 14 layers, 634 balls each, or 8876 balls

Total=18401 balls.

NOTE. Excellent solutions of problem 129 were received from H. C. Whitaker, P. S. Berg, G. B. M. Zerr, Martin Spinks, J. Scheffer, and O. S. Westcott. Mr. Gruber also furnished a non-rythmical solution. We think that his poetical solution is sufficiently clear and accurate as to be easily understood. The results of the various contributors differ slightly from Mr. Gruber's and from each other.

ALGEBRA.

106. Proposed by ELMER SCHUYLER, B. Sc., Professor of German and Mathematics, Boys' High School, Reading, Pa.

$$\frac{x^2+x}{y^2+y}=a\ ;\ \frac{x^2+y}{y^2+x}=b\ ;\ \text{find } x \text{ and } y.$$

I. Solution by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.; L. B. FILLMAN, Chester, Pa.; and the PROPOSER.

By composition and division we have at once

$$\frac{x^2+x+y^2+y}{x^2-y^2+x-y}=\frac{a+1}{a-1}\ldots(1),\ \frac{x^2+y+y^2+x}{x^2-y^2+y-x}=\frac{b+1}{b-1}\ldots(2).$$

Dividing (1) by (2) and removing the factor $x-y$, we get at once,

$$\frac{x+y-1}{x+y+1}=\frac{(b-1)(a+1)}{(b+1)(a-1)}.$$

Whence $x+y=\dfrac{ab-1}{a-b}$, and $y=\dfrac{ab-1}{a-b}-x$.

Substitute this value of y in either of the original equations and a quadratic results, which of course can always be solved.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa., and J. SCHEFFER, A. M., Hagerstown, Md.

Let $x=vy$.

$$\therefore\ y=\frac{a-v}{v^2-a}=\frac{vb-1}{v^2-b}.$$

$$\therefore\ v^3(b+1)-v^2(a+1)-bv(a+1)+a(b+1)=0.$$

$$\therefore\ v=1,\ v=\frac{a-b\pm\sqrt{[4a(b+1)^2+(a-b)^2]}}{2(b+1)}.$$

$v=1$ makes $x=y=-1$ which is so when $a=b=0$.
Let the second value of $v=m$.

$$\therefore\ y=\frac{a-m}{m^2-a},\ x=\frac{am-m^2}{m^2-a}.$$

III. Solution by J. W. YOUNG, Fellow and Assistant in Mathematics, Ohio State University, Columbus, O.

$$\frac{x^2+x}{y^2+y}=a.\ .\ .(1),\ \frac{x^2+y}{y^2+x}=b.\ .\ .(2).$$

Let $x'=x+1$, $y'=y+1$.

Then (1) becomes $\dfrac{xx'}{yy'}=a$, or $xx'=ayy'.\ .\ .(3)$.

(2) may be written

$$x^2+x+y+by=by^2+by+x+bx,\ \text{or}\ xx'+y(1+b)=byy'+x(1+b).$$

From (3) $\dfrac{(a-b)yy'}{1+b}=x-y=x'-y'$;

whence $x=y\left(y'\dfrac{a-b}{1+b}+1\right).\ .\ .(4)$, $x'=y'\left(y\dfrac{a-b}{1+b}+1\right).\ .\ .(5)$.

Multiplying (4) by (5),

$$xx'=yy'\left(y'\frac{a-b}{1+b}+1\right)\left(y\frac{a-b}{1+b}+1\right)=ayy'.\quad \text{From (3)}$$

$$\therefore yy'\left[\left(y'\frac{a-b}{1+b}+1\right)\left(y\frac{a-b}{1+b}+1\right)-a\right]=0.\ .\ .(6).$$

The first two factors $y=0$, $y'=0$, give the roots $y=0$, $y=-1$; whence $x=0$, $x=-1$.

The remaining factor in (6) is clearly a quadratic for y, which yields the remaining two roots.

IV. Solution by W. F. SHAW, 1600 Sabine Street, Austin, Tex.

Clear of fractions and subtract

$$\begin{array}{l} x^2+x=ay^2+ay \\ x^2+y=by^2+bx \\ \hline x-y=(a-b)y^2+ay-bx \end{array}$$

Multiply first by b, second by a, and subtract.

$$\begin{array}{l} bx^2+bx=aby^2+aby \\ ax^2+ay=aby^2+abx \\ \hline (a-b)x^2+ay-bx=ab(x-y) \\ (a-b)y^2+ay-bx=(x-y),\ \text{above results.} \\ \hline (a-b)(x^2-y^2)=(ab-1)(x-y) \\ x+y=[(ab-1)/(a-b)] \end{array}$$

Combining this with one of the first equations the values of x and y are found to be

$$x=\frac{a+b+2ab\pm\sqrt{[4a(b^2+b+1)+(a+b)^2]}}{2(a-b)},$$

$$y=\frac{-(a+b+2)\mp\sqrt{[4a(b^2+b+1)+(a+b)^2]}}{2(a-b)}.$$

Combining the first two equations to eliminate one of the letters, a cubic results. Two roots having been found the third is quickly seen to be -1. The other set of values is then, $x=-1$, $y=-1$.

Solved in substantially the same manner by *J. M. BOORMAN*, Woodmere, N. Y.